

Competitive Crossing Check for a 3% Determination of the Hubble Constant

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ABSTRACT

We show that a joint analysis involving independent cosmological probes at intermediate redshifts provides a remarkable cross-checking for the Hubble constant H_0 which is competitive with the existing measurements of the local Universe. Although dependent on the physics at $z \sim 1$, this result is fully independent on the calibrations involved in the cosmic distance ladder. The cosmological probes to be considered here are: (i) Angular diameter distances of galaxy clusters through SZE/X-ray technique ($0.14 \leq z \leq 0.89$), (ii) the ages of old galaxies at intermediate redshifts ($0.62 \leq z \leq 1.70$), (iii) measurements of the Hubble parameter $H(z)$ ($0.1 \leq z \leq 1.8$), and (iv) the baryon acoustic oscillation (BAO) signature ($z = 0.35$). By taking into account statistical plus systematic errors and assuming a flat Λ CDM cosmology (H_0 and Ω_M as free parameters), our joint analysis provides $H_0 = 73.4^{+3.1}_{-3.1}$ km s⁻¹ Mpc⁻¹ (1σ) when only the first three probes are combined. By adding the BAO scale, we obtain $H_0 = 74.1^{+2.2}_{-2.2}$ km s⁻¹ Mpc⁻¹ (1σ) which is a 3.0% determination of the Hubble constant at intermediate redshifts. Due to this special combination of tests, the present value of H_0 is competitive with the latest determinations based on nearby Cepheids and SNe Ia [Riess et al. ApJ 730, 119 (2001)]. This value can be much improved in the near future when more and larger samples (with smaller statistical and systematic uncertainties) become available. The present result also suggests that the method proposed here can be useful to achieve the wished theoretical and observational convergence on the value of H_0 .

Subject headings: Cosmological test, Sunyaev-Zeldovich effect, Galaxy ages, cosmic distances, Hubble constant

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1. Introduction

One of the most important observational quantities for cosmology is the Hubble constant, H_0 , whose value determines the present day expansion rate of the Universe. Eight decades after the original Hubble measurement, the determination of H_0 is still a very active subject (Tammann, Sandage & Reindl 2008; Mould & Sakai 2009; Freedman & Madore 2010; Tammann & Reindl 2011; Riess *et al.* 2009, 2011). Nowadays, several groups and missions are focusing their efforts on its precise determination since H_0 works like a key to quantify many astronomical phenomena in a wide range of scales involving galaxies, clusters, and superclusters. It also plays an important role for several cosmological calculations as the physical distances to objects, the age, size, and the matter-energy content of the Universe (Peebles 1993; Peacock 1999; Freedman *et al.* 2001; Jackson 2007).

Currently, the more robust constraints on H_0 are obtained from local tests ($z \ll 1$). The method is based on the cosmic *distance ladder* through a combination of different calibrators. The basic measurements and strategies commonly adopt: Cepheids, tip of the red giant branch, maser galaxies, surface brightness fluctuations, the Tully-Fisher relation, and type Ia supernovae (for a review see Freedman & Madore 2010). Along the last decade, Hubble Space Telescope (HST) Key Project did a magnificent job for decreasing significantly the errors on Hubble constant (Friedman and Madore 2010). Recently, Riess *et al.* (2011) also used the HST observations to determine H_0 from Cepheids and Supernovae (SNe Ia). Through a rigorous analysis of the statistical and systematic errors they obtained $H_0 = 73.8 \pm 2.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (1σ), corresponding to a 3.3% uncertainty. However, other authors also based on the HST observations have found $H_0 < 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from a different treatment. Probably, the main difficult of the methods based on the cosmic distance ladder is the possible existence of Hubble bubble or other local effects that would affect such measurements (Jha *et al.* 2007; Conley *et al.* 2007; Neill *et al.* 2007; Sinclair *et al.* 2010). The possibility of an observational convergence in the near future has increased in the last few years, however, additional progress, say, for a determination with 1% uncertainty will require to rebuild some aspects of the cosmic distance ladder (Suyu *et al.* 2012).

On the other hand, we know that the cosmologists desire accurate measurements of the H_0 mainly to refine the constraints on the neutrino masses ($\sum m_\nu$), density (Ω_Λ), and the equation of state parameter (ω) of dark energy based on CMB anisotropies data (Macri *et al.* 2006; Sekiguchi *et al.* 2010). Spergel *et al.* (2007) have shown that CMB studies can not supply strong constraints on the value of H_0 on their own. This problem occurs due to the high degree of degeneracy on the parameter space (Tegmark *et al.* 2004) since different grouping of the parameters (H_0 , Ω_M , Ω_Λ , ω , etc.) produce the same prediction of the cosmic microwave background radiation (CMB) anisotropies. It may be circumvented only by using

a prior on H_0 from independent measurements (Hu 2005). In this connection, Komatsu *et al.* (2011), used CMB, Supernovae (SNe Ia) and Baryon acoustic oscillations (BAO) data, to derive a model-dependent value of H_0 . It should be recalled that the phenomenology underlying the CMB test works at very high redshift ($z \simeq 1070$), during the decoupling between radiation and matter when the universe was only 370 million years old. Therefore, this determination of H_0 involves a combination of physics and phenomena at very different scales and epochs of the cosmic evolution (low, intermediate and high redshifts), and, more important to the present article, due to the inclusion of SNe Ia data, the derived value of H_0 is also somewhat dependent on the cosmic distance ladder. In such circumstances, it is particularly relevant to obtain accurate bounds on the value of H_0 from physics relying on many different kinds of observations, and, as far as possible, to avoid the combination of phenomena occurring at very different epochs (like SNe Ia and CMB).

In the last few years, some cosmological tests at intermediate redshifts ($z \sim 1$) have emerged as a promising technique to estimate H_0 (Simon *et al.* 2005; Deepak & Dev 2006; Cunha, Marassi & Lima 2007; Lima, Jesus & Cunha 2009; Stern *et al.* 2010; Busti *et al.* 2012). The main advantage of these methods is their independence of local calibrators (Carlstrom *et al.* 2002; Jones *et al.* 2005). In principle, such methods would provide a cross-checking for local direct estimates which are free from Hubble bubbles, since the majority of galaxy clusters are well inside the Hubble flow. However, all these methods are dependent on the assumptions about the astrophysical medium properties, as well as, of the cosmological model adopted in their analysis. It has also been argued that such probes, individually, are not yet competitive with the traditional methods based on the cosmic *distance ladder* (see, for instance, Freedman *et al.* 2001).

In this letter, we identify a remarkable complementarity involving four different cosmological probes at intermediate redshifts ($z \sim 1$) whose combination provides a valuable cross-checking for the Hubble constant value. The first test is based on measurements of the angular diameter distance from galaxy clusters via Sunyaev-Zeldovich effect (SZE) combined with measurements of the X-ray flux (SZE/X-ray technique). It was suggested long ago by many authors, but only recently it has been applied for a fairly large number of clusters (Bonamente *et al.* 2006; Cunha *et al.* 2007; Holanda *et al.* 2012a; 2012b). The second one is the age estimates of objects like galaxies and quasars at intermediate redshifts. It became possible through the new optical and infrared techniques together the recent advent of large telescopes (Alcaniz & Lima 1999; Lima & Alcaniz 2000; Alcaniz *et al.* 2003; Deepak & Dev 2006; Lima *et al.* 2009). The third possibility arises from measurements of the Hubble parameter, $H(z)$, from differential ages and baryon acoustic oscillations (Simon *et al.* 2005; Gaztañaga *et al.* 2009; Stern *et al.* 2010). As we shall see, the cooperative interaction among these three independent probes already reduce greatly the errors on the Hubble constant,

and, more interesting, located just on a redshift zone distinct both from CMB anisotropies and the methods defined by the cosmic distance ladder (Cepheid, SNe Ia, etc.). In principle, it is possible to derive tighter constraints on the possible values of H_0 once the cosmology is fixed. However, in our analysis we marginalize over the cosmology by adding the fourth test which is represented by the BAO signature. The advantage of a combination with BAO (a scale independent of H_0) was suggested earlier by Cunha, Marassi & Lima (2007), however, restricted only to the Hubble constant determination based on the SZE technique. In what follows, we show that the distinct probes adopted here act in concert to predict tighter constraints on the value of H_0 .

2. Basic Equations, Probes and Samples

Let us now consider that the Universe is described by a flat Friedmann-Robertson-Walker (FRW) geometry (in our units $c = 1$)

$$ds^2 = dt^2 - a^2(t) [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi)], \quad (1)$$

whose evolution is driven by cold dark matter plus a cosmological constant (Λ CDM).

In this background, the angular diameter distance (ADD), \mathcal{D}_A , can be written as (Lima & Alcaniz 2002; Lima *et al.* 2003; Holanda *et al.* 2010)

$$\mathcal{D}_A(z; h, \Omega_M) = \frac{3000h^{-1}}{(1+z)} \int_0^z \frac{dz'}{\mathcal{H}(z'; \Omega_M)} \text{Mpc}, \quad (2)$$

where $h = H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (subscript 0 denotes present day quantities), and the dimensionless function $\mathcal{H}(z'; \Omega_M)$ is given by $\mathcal{H} = [\Omega_M(1+z')^3 + (1 - \Omega_M)]^{1/2}$.

Bonamente and collaborators (2006) determined the ADD distance to 38 galaxy clusters in the redshift range $0.14 \leq z \leq 0.89$ using X-ray data from Chandra and Sunyaev-Zeldovich effect data from the Owens Valley Radio Observatory and the Berkeley-Illinois-Maryland Association interferometric arrays. Assuming spherical symmetry, the cluster plasma and dark matter distributions were analyzed by using a hydrostatic equilibrium model accounting for radial variations in density, temperature and abundances. The common statistical contributions for this galaxy clusters sample are: SZE point sources $\pm 8\%$, X-ray background $\pm 2\%$, Galactic $N_H \leq \pm 1\%$, $\pm 15\%$ for cluster asphericity, $\pm 8\%$ kinetic SZ and for CMB anisotropy $\leq \pm 2\%$. Estimates for systematic effects are as follow: SZ calibration $\pm 8\%$, X-ray flux calibration $\pm 5\%$, radio halos $+3\%$ and X-ray temperature calibration $\pm 7.5\%$. Typical statistical errors amounts for nearly 20% in agreement with other works (Mason *et*

al. 2001; Reese *et al.* 2002, 2004), while for systematics we also find typical errors around + 12.4% and - 12% (see table 3 in Bonamente *et al.* 2006).

On the other hand, the age-redshift relation, $t(z)$, for a flat (Λ CDM) model has also only two free parameters (H_0, Ω_M)

$$t(z; h, \Omega_M) = H_0^{-1} \int_0^{\frac{1}{1+z}} \frac{dx}{x \sqrt{\Omega_M x^{-3} + (1 - \Omega_M)}}. \quad (3)$$

Note that for $\Omega_M = 1$ the above expression reduces to the well known result for Einstein-de Sitter model (CDM, $\Omega_M = 1$) for which $t(z) = \frac{2}{3} H_0^{-1} (1+z)^{-3/2}$. As one may conclude from the above equation, limits on the cosmological parameters Ω_M and H_0 (or equivalently h), can be derived by fixing $t(z)$ from observations. Note also that the age parameter, $T = H_0 t_z$, depends only on the product of the two quantities H_0 and t_z , which are usually estimated from completely independent methods (Alcaniz & Lima 1999; Lima & Alcaniz 2000; Cunha & Santos 2004; Friaça *et al.* 2005).

In this context, Ferreras *et al.* (2009) catalogued 228 red galaxies on the interval $0.4 < z < 1.3$ using HST/ACS slitless grism spectra from the PEARS program thereby studying the stellar populations of morphologically selected early-type galaxies from GOODS North and South fields. The first subsample adopted here consists of six passively evolving old red galaxies selected from the original Ferreras *et al.* sample. The second data set of old objects is also a subsample of the Longhetti *et al.* (2007) sample which is composed by nine field galaxies spectroscopically classified as early-types at $1.2 < z < 1.7$. They result from a near-IR spectroscopic follow-up of a complete sample of bright early red galaxies selected from the Munich Near-IR Cluster Survey (MUNICS) that provides optical (B, V, R, I) and near-IR (J and K') photometry. The galaxy ages were estimated by making use of stellar populations with the degeneracy between the age of the best-fitting models and the star formation time scale being broken through a mass-weighting of the masses. For both samples, the selected data set (eleven galaxies) provide the most accurate ages and the most restrictive galaxy ages.

In Figure 1a, we display the sample constituted by the eleven data points chosen from two distinct subsamples of old objects. As discussed by Lima *et al.* (2009), we have added an incubation time with a conservative error bar for all galaxies. Such a quantity is an estimate of the amount of time interval from the beginning of structure formation process in the Universe until the formation time (t_f) of the object itself (the time of the radiation era is negligible). It is also assumed that t_{inc} varies slowly with the galaxy and redshift in our sample, and, in order to account for our ignorance about this kind of “nuisance” parameter (Fowler 1987; Sandage 1993), we have associated a reasonable uncertainty, $\sigma_{t_{inc}}$. In what

follows, we consider that $t_{inc} = 0.8 \pm 0.4$ Gyr. We also combine statistical and systematic errors (for more details, see Lima *et al.* 2009).

In Figs. 1b, (1c), we compare the age of these old objects at intermediate redshifts with the predictions of the Λ CDM models for different values of the free parameters (Ω_M and h). It is also worth noticing that the present status of systematic uncertainties in this context is still under debate. Jimenez *et al.* (2004) studied sources of systematic errors in deriving the age of a single stellar population and concluded that they are not larger than 10% – 15% per cent. Others authors consider systematic errors around 20% (Percival & Salaris 2009). In the present study we have adopted 15% for all data.

The third observational probe comes from $H(z)$ data, obtained from differential ages of galaxies and radial BAO (Simon *et al.* 2005; Gaztañaga *et al.* 2009). Some years ago, Jimenez and collaborators suggested an independent estimator for the Hubble parameter (differential ages of galaxies) and used it to constrain the equation of state of dark energy (Jimenez *et al.* 2002, 2003). Later on, the differential ages of passively-evolving galaxies were used to obtain $H(z)$ in the range of $0.1 \lesssim z \lesssim 1.8$ (Simon *et al.* 2005), and this sample was further enlarged by Stern *et al.* (2010). In addition, Gaztañaga *et al.* (2009) took the BAO scale as a standard ruler in the radial direction (Peak Method) thereby obtaining three more additional data: $H(z = 0.24) = 79.7 \pm 2.7$, $H(z = 0.34) = 83.9 \pm 3.2$, and $H(z = 0.43) = 86.5 \pm 3.5$, which are model and scale independent. Now, by comparing the theoretical expression, $H(z) = H_0 [\Omega_M(1+z)^3 + (1 - \Omega_M)]^{1/2}$, with the observational data the corresponding bounds can be readily derived. We remark that the relative age difference (the key to the method) is only of the order of 2 – 3% (Stern *et al.* 2010). However, to be more conservative we are assuming here systematic errors of 8% for the relative age difference and radial BAO data.

As we shall see next, a joint analysis based on the above 3 different probes already provides tight constraints on the value of H_0 . However, as shown by Cunha *et al.* (2007), the analysis of \mathcal{D}_A data (from SZE/X-ray technique) leads to more stringent constraints on the space parameter (Ω_M, h) when combined with the BAO signature (Eisenstein *et al.* 2005; Percival *et al.* 2010), and the same happens when the ages of high redshift objects are considered (Lima *et al.* 2009). Therefore, instead to fix a definite flat Λ CDM cosmology (for a given value of Ω_M), it is natural to leave Ω_M free which will be more accurately fixed by adding the BAO signature as a fourth probe to the complete joint analysis performed here.

It is widely known that the BAO peak (detected from a sample of 46748 luminous red galaxies selected from the SDSS Main Sample) is predicted to arise precisely at the measured scale of $100 h^{-1}$ Mpc. It is interpreted as a consequence of the baryon acoustic oscillations in the primordial baryon-photon plasma prior to recombination. Such a measurement can

be characterized by the dimensionless parameter

$$\mathcal{A} \equiv \frac{\Omega_M^{1/2}}{\mathcal{H}(z_*)^{1/3}} \left[\frac{1}{z_*} \Gamma(z_*) \right]^{2/3} = 0.469 \pm 0.017, \quad (4)$$

where $z_* = 0.35$ is the redshift at which the acoustic scale has been measured, and $\Gamma(z_*)$ is the dimensionless comoving distance to z_* . Note that the quantity given by (4) is independent of the Hubble constant and, as such, the BAO signature alone constrains only the Ω_M parameter. This property is very characteristic of the BAO signature thereby differentiating it from many others classical cosmological tests.

3. Complementarity for H_0

To begin with, let us consider a joint analysis based only on the combination of the first three probes, namely: (i) SZE/X-ray distances, (ii) the age of the oldest intermediate redshift objects, and (iii) the measurements of the Hubble Parameter $H(z)$. Further, a complete joint analysis including the BAO signature from the SDSS catalog it will be performed. For both analysis, we stress that a specific flat Λ CDM cosmology has not a priori been fixed.

All the observational expression adopted here have only two free parameters (h, Ω_M) . In this way, we perform the χ^2 statistics over the $\Omega_M - h$ plane. Combining the three tests discussed above the χ^2 **value reads**:

$$\chi^2(z|\mathbf{p}) = \sum_i \frac{(\mathcal{D}_A(z_i; \mathbf{p}) - \mathcal{D}_{Aobs,i})^2}{\sigma_{\mathcal{D}_{Aobs,i}}^2 + \sigma_{stat}^2 + \sigma_{syst}^2} + \sum_j \frac{(t(z_j; \mathbf{p}) - t_{inc} - t_{obs,j})^2}{\sigma_{t_{obs,j}}^2 + \sigma_{t_{inc}}^2 + \sigma_{syst}^2} + \sum_k \frac{(H(z_k; \mathbf{p}) - H_{obs,k})^2}{\sigma_{H_{obs,k}}^2 + \sigma_{syst}^2}, \quad (5)$$

where the quantities with subindex “obs” are the observational quantities, $\sigma_{\mathcal{D}_{Ao,i}}$ is the uncertainty in the individual distance, σ_{stat} is the contribution of the statistical errors, $\sigma_{t_{inc}}$ is the incubation time error, σ_{syst} are the contribution of the systematic errors for each sample added in quadrature and the complete set of parameters is given by $\mathbf{p} \equiv (h, \Omega_M)$.

In Fig. 2a we show dimensionless Hubble constant (h), versus the the matter density parameter (Ω_M). Our analysis combining statistical and systematic errors from galaxy ages are in black lines with 68.3%, 95.4% and 99.7% confidence levels (c.l.). In the same way, the green lines represent the confidence levels for the SZE/X-ray sample whereas the corresponding constraints from Hubble parameter $H(z)$ sample are show in orange lines. As should be expected, the constraints of each sample on (h, Ω_M) plane are too weak when individually considered. However, due to the complementarity among them, our joint analysis for these three samples predicts $H_0 = 73.4 \pm 4.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (6.4%) and $\Omega_M = 0.290_{-0.061}^{+0.078}$ (1 σ c.l.)

for two free parameters with $\chi_{min}^2 = 41.42$. The reduced values are $\chi_{red}^2 = 0.702$ (including systematics) and $\chi_{red}^2 \cong 1$ (no systematics).

As remarked earlier, in order to put tighter constraints on the parameters h and Ω_M we must add the BAO signature which is independent of the Hubble parameter and, as such, can break the degeneracy on the mass density parameter (Cunha *et al.* 2007, Lima *et al.* 2009).

In Fig. 2b, we display the contours on the space parameter obtained through a joint analysis involving the combination of all four probes. The dashed lines are cuts on (h, Ω_M) plane from BAO signature. The red, green and blue contours are constraints with 68.3%, 95.4% and 99.7% c.l., respectively. This complete analysis provides $H_0 = 74.1_{-3.3}^{+3.3} \text{ km.s}^{-1}.\text{Mpc}^{-1}$ (4.5% uncertainty) whereas the density parameter is $\Omega_M = 0.278_{-0.028}^{+0.034}$ for two free parameters with a $\chi_{min}^2 = 41.53$ ($\chi_{red}^2 = 0.704$).

In Figure 1c, we show the likelihood function for the h parameter in a flat Λ CDM universe. Both curves were obtained by marginalizing on the matter density parameter (Ω_M). The shadow lines are cuts in the regions of 68.3% and 95.4% probability. For the solid black line the BAO signature was not considered. The constraints for the black line are $h = 0.734 \pm 0.031$ (0.064) with 1σ (2σ), respectively. For the red line the BAO signature **has been included**. The constraints are $h = 0.741 \pm 0.022$ (corresponding to 3% error in h) and 0.045 (6.1%) with 1σ and 2σ , respectively. For all these analyses, the statistical and systematic errors were added.

In table 1, we compare the results derived here with the latest determinations of the Hubble parameter. Note that competitive constraints were obtained using only probes of intermediate redshifts, and, perhaps, more interesting, the present constraints are consistent with recent results provided by independent methods at different epochs of the cosmic evolution.

Table 1: Limits to h for **different methods and epochs**

Reference	Method	h (1σ)	Epoch
Komatsu <i>et al.</i> 2009 ...	CMB	0.705 ± 0.013	$z \sim 1100$
Freedman & Madore 2010...	Cepheid Variables	0.730 ± 0.045	$z \simeq 0$
Riess <i>et al.</i> 2011...	Cepheid+SNe+Maser	0.738 ± 0.024	$z \simeq 0$
This paper...	SZE/X-ray+Age+H(z)	0.734 ± 0.31	$0.1 < z < 1.8$
This paper...	SZE/X-ray+Age+H(z)+BAO	0.741 ± 0.022	$0.1 < z < 1.8$

4. Conclusions

Several ongoing and future experiments (HST Key Project, SH_0ES , PLANCK, Spitzer, GAIA, JWST, and others) are dedicated to measure more precisely the present day value of the Hubble constant. Numerous fields of research in Astronomy will benefit with the planned improvement for the 2% measurement of H_0 likely to be achieved in the present decade.

In this letter, we have demonstrated that a joint analysis involving four independent cosmological tests at intermediate redshifts provides a 3% determination of H_0 including statistical plus systematic errors. Our approach was separated in two distinct parts. Firstly, we have implemented a joint analysis involving only three probes, namely: (i) angular diameter distance from galaxy clusters (SZE/X-ray technique), (ii) a sample of old objects at intermediate redshifts, and (iii) measurements of the Hubble parameter $H(z)$ (at $z \neq 0$). This analysis has shown that such tests act in concert thereby predicting an unexpected tight value of H_0 . Secondly, we have added the BAO signature which is independent of the H_0 thereby obtaining a more accurate value of the Hubble constant.

It should be stressed that our 3.0% determination of H_0 became possible only when the BAO signature was implemented in our joint analysis. In the framework of a flat Λ CDM model we have obtained $h = 0.739 \pm 0.022$ (including statistical plus systematic uncertainties). As shown in table I, this value is not only consistent, but has the same precision of a recent H_0 determination using nearby Cepheids and Supernovae (Riess *et al.* 2011). It is advocated here that the present determination of the Hubble constant based only on probes at intermediate redshifts provides a competitive-crossing checking for any determination of H_0 . We also believe that the method presented here has a potential at least comparable to other independent approaches. The main advantage of the present treatment is that it does not rely on extragalactic distance ladder being fully independent of any local calibrator. Naturally, its basic disadvantage rests on the large and different systematic uncertainties appearing in each probe when separately applied. However, the remarkable complementarity among the four tests works in concert thereby reducing greatly the possible degeneracy and uncertainties appearing in the present determination of the Hubble constant.

Acknowledgments

JASL is partially supported by CNPq and FAPESP (Brazilian Research Agencies).

REFERENCES

- Alcaniz J. S., Lima J. A. S., 1999, ApJ, 521, L87, preprint (astro-ph/9902298)
- Alcaniz J. S., Lima J. A. S., Cunha, J. V., 2003, MNRAS, 340, L39, preprint (astro-ph/0301226)
- Bonamente M. *et al.*, 2006, ApJ, 647, 25
- Busti V. C., Guimarães R. N., Lima, J. A. S., preprint (arXiv:1201.1260[astro-ph.CO])
- Carlstrom, J. E., Holder, G. P., Reese, E. D., 2002, ARA&A, 40, 643
- Conley A. *et al.*, 2007, ApJ, 664, L13
- Cunha J. V., Santos, R. C., 2004, Int. J. Mod. Phys. D, 13, 1321, preprint (arXiv:astro-ph/0402169)
- Cunha J. V., Marassi L., Lima J. A. S., 2007, MNRAS, 379, L1, preprint (astro-ph/0611934)
- Deepak J., Dev, A., 2006, Phys. Lett. B, 633, 436
- Eisenstein D. J. *et al.*, 2005, ApJ, 633, 560
- Ferreras I., Pasquali A., Malhotra S. *et al.*, 2009, ApJ, 706, 158
- Fowler W. A., 1987, QJRAS, 28, 87
- Freedman W. L. *et al.*, 2001, ApJ, 553, 47
- Freedman W. L., Madore B. F., 2010, Annu. Rev. Astro. Astrophys., 48, 673
- Friça A. C. S., Alcaniz J. S., Lima J. A. S., 2005, MNRAS, 362, 1295
- Gaztañaga E., Cabré A., Hui, L., 2009, MNRAS, 399, 1663
- Holanda R. F. L., Lima J. A. S., Birro M. R., 2010, ApJL, 722, L233, preprint (arXiv:1005.4458)
- Holanda R. F. L., Cunha J. V., Marassi L., Lima J. A. S., 2012, JCAP, 1202, 035, preprint (arXiv:1006.4200 [astro-ph.CO])
- Holanda R. F. L., Cunha J. V., Lima, J. A. S., 2012, Gen. Relativ. Gravit., 44, 501, preprint (arXiv:0807.0647)

- Hu W., 2005, in ASP Conf. Ser. 339: Observing Dark Energy, ed. S. C. Wolf & T. R. Lauer, p. 215
- Jackson N., 2007, *Living Rev. Relativ.*, 10, 4
- Jha S., Riess A. G., Kirshner R. P., 2007, *ApJ*, 659, 122
- Jimenez R., Loeb A., 2002, *ApJ*, 573, 37
- Jimenez R. *et al.*, 2003, *ApJ*, 593, 622
- Jimenez R. *et al.*, 2004, *MNRAS*, 349, 240
- Jones M. E. *et al.*, 2005, *MNRAS*, 357, 518
- Komatsu E. *et al.*, 2009, *ApJS*, 180, 330 (WMAP collaboration)
- Komatsu E. *et al.*, 2011, *ApJS*, 192, 18 (WMAP collaboration)
- Lima J. A. S., Alcaniz J. S., 2000, *MNRAS*, 317, 893, preprint (arXiv:astro-ph/0005441)
- Lima J. A. S., Alcaniz J. S., 2002, *ApJ*, 566, 15, preprint (astro-ph/0109047)
- Lima J. A. S., Cunha J. V., Alcaniz J. S., 2003, *Phys. Rev. D*, 68, 023510, preprint (arXiv:astro-ph/0303388)
- Lima J. A. S., Jesus J. F., Cunha J. V., 2009, *ApJ*, 690, L85, preprint (arXiv:0709.2195)
- Longhetti M., Saracco P., Severgnini P. *et al.*, 2007, *MNRAS*, 374, 614
- Macri L. M. *et al.*, 2006, *ApJ*, 652, 1133
- Mason B. S. *et al.*, 2001, *ApJ*, 555, L11
- Mould J., Sakai S., 2009, *ApJ*, 697, 996
- Neill J. D., Hudson M. J., Conley A., 2007, *ApJ*, 661, L123
- Peacock J. A., 1999, *Cosmological Physics*. Cambridge Univ. Press, Cambridge
- Peebles P. J. E., 1993, *Principles of Physical Cosmology*. Princeton Univ. Press, Princeton, NJ
- Percival S. M., Salaris M., 2009, *ApJ*, 703, 1123
- Percival S. M. *et al.*, 2010, *MNRAS*, 401, 2148

- Reese E. D. *et al.*, 2002, ApJ, 581, 53
- Reese E. D., 2004, in Measuring and Modeling the Universe, ed. W. L. Freedman (CUP) p. 138
- Riess A. G. *et al.*, 2009, ApJ, 699, 539
- Riess A. G. *et al.*, 2011, ApJ, 730, 119
- Sandage A., 1993, Astron. J., 106, 719
- Sandage, A., Tamman, G. A., Saha, A., Reindl, B., Machetto, F. D. & Panagia, N. 2006, ApJ, 653, 843
- Sekiguchi T. *et al.*, 2010, J. Cosmology Astropart. Phys., 03, 15
- Simon J., Verde L., Jimenez R., 2005, Phys. Rev. D, 71, 123001
- Sinclair B., Davis T. M., Haugblle T., 2010, ApJ, 718, 1445
- Spergel D. N. *et al.*, 2007, ApJ, 170, 377
- Stern D. *et al.*, 2010, J. Cosmology Astropart. Phys., 02, 008
- Suyu S. H. *et al.*, 2012, in The Hubble Constant: Current and Future Challenges ed. Sherry H. Suyu, Tommaso Treu, Roger D. Blandford, Wendy L. Freedman, preprint (arXiv:1202.4459)
- Tammann G., Sandage A., Reindl B., 2008, ApJ, 679, 52
- Tammann G. A., Reindl B., 2011, preprint (arXiv:1112.0439)
- Tegmark M. *et al.*, 2004, Phys. Rev. D, 69, 103501
- Wang S., Li X.-D., Li M., 2010a, Phys. Rev. D, 82, 103006
- Wang S. *et al.*, 2010b, Astron. J., 139, 1438

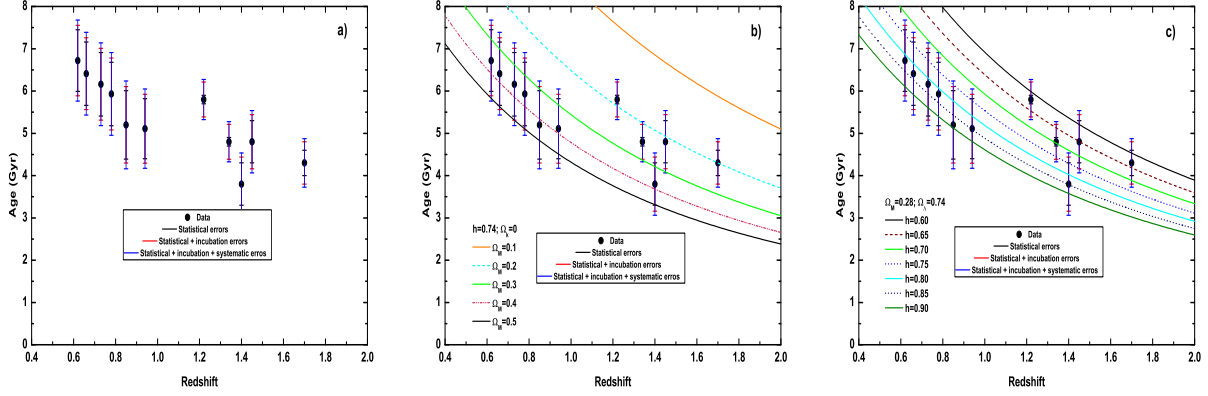


Fig. 1.— Old galaxies data at intermediate redshifts as a cosmic probe. **a)** The age-redshift plane and the total sample of galaxies. Black points for $z < 1.0$ and $z > 1.2$ correspond, respectively, to the Ferreras *et al.* (2009) and Longhetti *et al.* (2007) samples. The black, red and blue bars represent the statistical, statistical+incubation and statistical+incubation+systematic errors, respectively. These data points correspond to the 11 galaxies of our selected subsample (see the main text). **b)** Age-redshift relation (the effect of Ω_M). The age of the Universe for some selected values of the density parameter. All plots were drawn by choosing the h parameter as the best fit of our analysis ($h = 0.74$). For smaller values of Ω_M the ages of the Universe for a given redshift increase, thereby accommodating the oldest selected objects. **c)** Age-redshift relation (the h effect). Dotted curves are the predictions of the cosmic concordance model ($\Omega_M = 0.28, \Omega_\Lambda = 0.7$) and different values of h . We see that for values smaller than $h = 0.6$ or bigger than $h = 0.9$ the curves predicted by the models move away from the data.

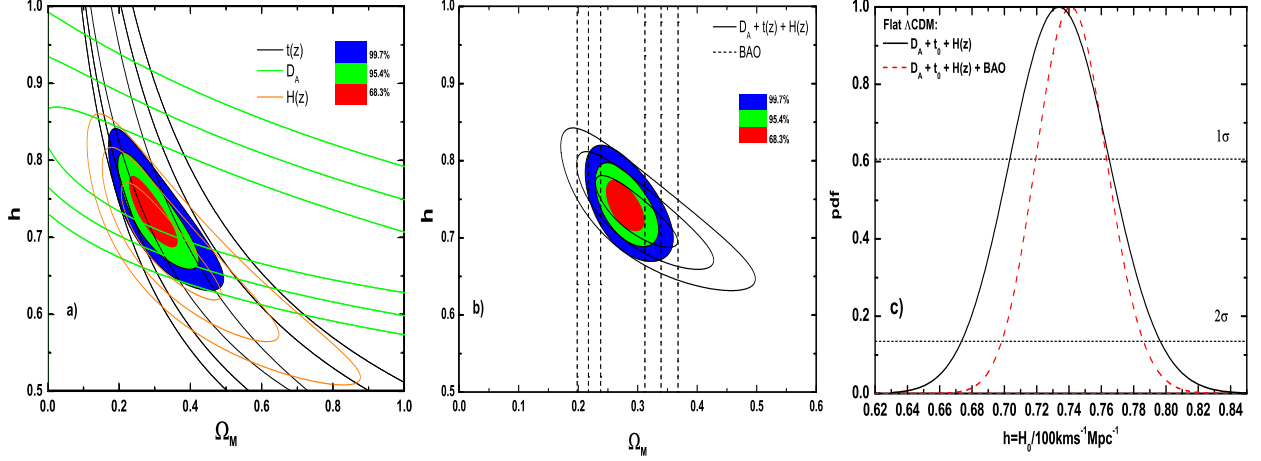


Fig. 2.— The determination of H_0 at intermediate redshifts. **a)** H_0 and the complementarity of three different probes. Confidence regions (68.3%, 95.4% and 99.7%) in the (Ω_M, h) plane provided by the SZE/X-ray + galaxy age + $H(z)$ data including statistical plus systematic errors. The best-fitting values are $h = 0.734$ and $\Omega_M = 0.290$. **b)** The BAO razor. Contours in the $\Omega_M - h$ plane from a joint analysis involving the SZE/X-ray + galaxy age + $H(z)$ + BAO data. The contours correspond to 1, 2, 3 σ confidence levels. The best-fitting model converges to $h = 0.741$ and $\Omega_M = 0.278$. **c)** The final values of H_0 . Likelihood functions for the h parameter in a flat Λ CDM Universe. As indicated in the figure, the dashed (red) curve correspond to a joint analyses involving from SZE/X-ray + galaxy age + Hubble parameter whereas the solid black curve includes the BAO signature. The horizontal lines are cuts in the regions of 68.3% and 95.4% probability. By including statistical plus systematic errors the joint analysis performed with the four different probes provides $h = 0.741 \pm 0.022$ (1 σ). As can be seen from Table 1, this constraint on H_0 is in agreement with the latest independent determinations of the Hubble constant (Freedman and Madore 2010; Komatsu *et al.* 2009; Riess *et al.* 2011).